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Scheduling in a Single-stage, Multi-item Compatible Process using Multiple Arc Network with Gains Model

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ABSTRACT

The problem of scheduling a given set of machines in a single-stage, multi-item compatible environment, with the objective of maximizing capacity utilization has been formulated [1] as a maximal flow problem in a Multiple Arc Network (MAN). The model is based on the quantities of inputs and outputs at the processing unit. Further, the inherent assumption of the model is that the input and output quantities are identical. In reality, while processing most of the products, they not only undergo transformation but also the output quantities or weights will differ. Hence the scheduling problem has been formulated as a maximal flow problem in a MAN with gains. The gain represents the factor which brings about change in quantity, such as yield. The model aims to provide optimal production schedule with an objective of maximizing capacity utilization, so that the customer-wise delivery schedules are fulfilled, keeping in view the customer priorities. Algorithms have been presented for solving the MAN formulation of the production planning with customer priorities as well as the gain factor. The suitability of the algorithms has been demonstrated with an example.

Keywords: *Scheduling, Maximal Flow Problem, Multiple Arc Network with Gains Model, Optimization.*

INTRODUCTION

It is common to find production systems characterized by having a single stage or a previously identified bottleneck stage [2]. Further, the production planning process usually includes a stage prior to the planning horizon, in which the various orders of a product are grouped and, after checking the inventory status, a "total production demand" is determined for each product. Products may have intermediate due dates, i.e. some partial or total demand for a particular product may have more than one specific due date within the planning horizon for either the same or different customers.

For example, it can be a workshop having a number of lathes which are being used for manufacturing various types of roller sets for a steel mill or a chemical industry. A basic feature of chemical industry production process is that it involves multiple stages but there is a main bottleneck stage that uses the most expensive operating resources of the plant.

This fact forces the other stages of the process to subordinate to these units, and allows the simplification of the scheduling process to the single bottleneck stage as in [3]. It also requires the most efficient use of these units by using them at their maximum load and by minimizing setup times.

A general overview of many different aspects of production planning and inventory management can be found in [4] and in standard textbooks such as [5, 6, 7]. Scheduling problems in single-stage and continuous multiproduct processes on parallel lines with intermediate due dates and especially restrictions on minimum run lengths has been considered in [8]. A multi-period mixed integer linear programming model has been envisaged for the simultaneous planning and scheduling of single stage multi-product continuous plants with parallel units in [9]. The problem of scheduling a single-stage multi-product batch chemical process with fixed batch sizes is present as a mixed-integer nonlinear programming model [10] to determine the schedule of batches, the batch size, and the number of overtime shifts that satisfy the demand at minimum cost for this process. Mixed integer linear programming has been considered [11] for the short-term scheduling of a single-stage batch edible-oil deodorizer that can process multiple products in several product groups. Scheduling and optimization for a class of single-stage hybrid manufacturing systems has been present by [12]. A heuristic Solution for Scheduling Single Stage Parallel Machines Production of Calcium Silicate Masonry Units with Sequence-Dependent Changeover Times to Improve Energy Efficiency is presented in [13]. The problem of allocating the limited capital resources to the various stages of a multistage production system, in order to improve the yield of the production stages and at the same time minimize the annual cost has been considered in [14].

Since the pioneering work of Ford and Fulkerson in 1962 [15], the use of network models and algorithms have proved to be particularly successful in different application areas. A Multiple Arc Network (MAN) model of production planning in a steel mill is presented in [16]. The problem of scheduling a given set of equipments in a single-stage, multi-item compatible environment, with the objective of maximizing capacity utilization has been formulated as a maximal flow problem in a Multiple Arc Network (MAN) in [17].

To demonstrate the practical application of excel solver, the working implementation of the MAN model formulated in [17], using excel solver were presented in [18]. In all these models, the quantum of input and output at each of the node were considered same. However it might not be the case in practice. The output quantities will be affected by the yield factor at the processing stage.

In this paper, the scheduling problem has been formulated as a maximal flow problem in a MAN with gains. The gain represents the factor which brings about change in quantity, such as yield. The model aims to provide optimal production schedule with an objective of maximizing capacity utilization, so that the customer-wise delivery schedules are fulfilled. , keeping in view the customer priorities.

THE PROBLEM

We consider the problem of finding optimal production plan for a production process which can handle different types of products, where changeover for handling one type of product to the other type incurs certain costs. For example, we can consider the production process in an integrated steel plant, which can produce different grades and sections of steel products through different routes of production, by suitably changing the sequences.

We shall consider a sequence to enable production of one or more products. The capacity of a stage is determined by the upper limit for the quantity that can be processed for each of the products in a sequence. These changeover costs increase with the number of changeovers and hence to minimize the costs associated with the product changeover, the planning should be such that similar types of products should be processed successively so that the total number of changeovers and in turn the associated costs are minimized. The problem of cost minimization is equivalent to the problem of minimizing the number of changeovers or equivalently maximizing the capacity utilization in between every changeover or maximizing the total capacity utilization.

Further, certain planning procedures call for priority planning where different customers are assigned different priorities.The problem of production planning can be formulated into MAN with gains models and can be solved sequentially using the following algorithms:

Algorithm 1: Algorithm for maximizing flow along a MAN with gains.

Algorithm 2: Algorithm for maximizing flow along a MAN with gains with priority arcs.

MULTIPLE ARC NETWORK (MAN)

Consider the flow network N=(s,t,V,A,b) with the digraph (V,A) together with a source sεV with 0 indegree, a sink or terminal tεV with 0 outdegree with |V| vertices or nodes.

A has as elements subsets of V of cardinality two called arcs together with the arc number i and with |A| arcs, i=0,1,2,3,….,n where i denotes the arc number.

We use the following notations:

(x,i,y) = i^{th} directed arc from the vertex x to the vertex y.

b(x,i,y) ε Z^+ for each (x,i,y)εA is the bound or capacity of the arc (x,i,y).

$A_i(x) = \{y\varepsilon V/(x,i,y)\varepsilon A\}$

$B_i(x) = \{y\varepsilon V/(y,i,x)\varepsilon A\}$

We assume that arcs of the form (x,i,x) for xεV do not exist in A.

Define $A(x) = \bigcup_i A_i(x)$ and $B(x) = \bigcup_i B_i(x)$

A flow f in N is a vector in $R^{|A|}$, one component f(x,i,y) for each arc (x,i,y)εA.

Let $f(x,-,y) = \sum_i f(x,i,y)$

Definition

A static flow of value v from source s to sink t in the network N is a function f from A to non-negative reals such that the following equations and inequalities are satisfied.

$$\sum_{y \varepsilon A(x)} f(x,-,y) - \sum_{y \varepsilon B(x)} f(y,-,x) = \begin{cases} v(f) & \text{if } x=s \\ 0 & \text{if } x \neq s,t \\ -v(f) & \text{if } x=t \end{cases} \quad (1)$$

$$f(x,i,y) \leq b(x,i,y) \ \forall (x,i,y) \ \varepsilon A \quad (2)$$

$$f(x,i,y) \geq 0 \ \forall (x,i,y) \ \varepsilon A \quad (3)$$

The static flow problem is to maximize the flow from s to t such that the flow f satisfies (1), (2), and (3).

MAN with Priority Arcs

Consider the MAN N=(s,t,V,A,b). The flow in the network N is characterized by permeability of flow from source s to sink t only through arcs with like priority numbers. For any arc (x,i,y) εA, let the arc number i denote the priority number as well.

METHODOLOGY

The problem of production planning can be formulated into MAN with gains model. We consider the following notations:

Customer C_i i=1,2,…..n

Products S_k k=1,2,…..l

Equipments E_r r=1,2,…..p

Processing Stages s s=1,2,…..v

Demand for the product S_k by customer C_i Dki

Delivery time for the product S_k of customer C_i T(k,i)

Capacity of processing theproduct S_k in the sequence E_r A_{kr}

Processing time for the product Sk of customer Ci tki

Yield rate for the product Sk gk

Methodology for determining the periods for process commencement

Consider the MAN N=(s,t,V,A,b) with v=|V| vertices, representing the processing stage (equipments) and products. The arcs of A include all possible customer-wise orders and all possible products that can be processed on equipments. This network will have a specific structure for a given processing unit. This network will be referred to as the PROCESSNET. A typical PROCESSNET for a process consisting of "p" similar equipments capable of processing "l" products, in a single stage, is shown in Figure 1. Periods for the process commencement is given by To=T(k,i)-tki.

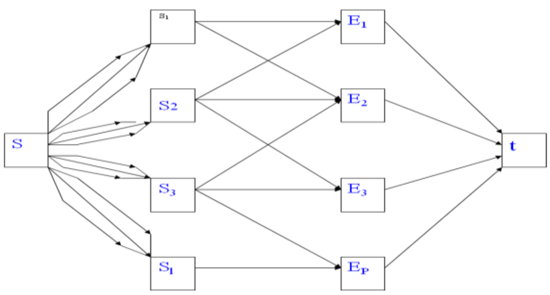

Fig1. *A typical PROCESSNET*

Methodology for Maximizing Flow along A MAN with Gains

The labeling algorithm for maximizing flow along a MAN with gains where the gain factor along the arc (x,i,y) εA of the MAN N=(s,t,V,A,b) is denoted by g(x,i,y) is given below:

Algorithm 1

Step1: Start with flow f≡0 as the feasible flow with v(f)=v(0)=0.

Go to Routine A.

Routine A

Step A1: Label s as (-,-,∞).

Step A2: Take any vertex x from the set of vertices which are labeled and unscanned through.

Step A3: If (x,i,y) εA and f(x,i,y)<b(x,i,y), label y as $(x^+,i,\varepsilon(y))$, where

ε(y)=Min.{ε(x)g(x,i,y),(b(x,i,y)-f(x,i,y))g(x,i,y)}

If (y,i,x) εA and f(y,i,x)>0, label y as (x^-,i,ε(y)), where

ε(y)=Min.{ ε(x)/g(y,i,x), f(y,i,x)/g(y,i,x)}

Repeat this step until no further vertices can be labeled or t is labeled.

StepA4: If t is not labeled, it suggests that there is no flow augmenting path from s to t, and hence f is the maximal flow.

If t is labeled, go to Routine B.

Routine B

StepB1: If t has label (x^+,i,ε(y)), determine f'(x,i,t) = f(x,i,t)+ ε(t)/g(x,i,t).

If t has label (x^-,i,ε(y)), determine f'(t,i,x) = f(t,i,x)- ε(t)g(t,i,x).

Step B2: In general, at vertex x_i with path of labeled vertices as

x_0=s, x_1, x_2, ….., x_i,…..,x_n, x_{n+1}=t and if x_i has label (x^+_{i-1},i,ε(x_i)), determine

f'(x_{i-1},i,x_i) = f(x_{i-1},i,x_i) + {ε(t) $\Pi_{j=i}^{n+1}$g(x_j,i,x_{j-1})}/ $\Pi_{j=i}^{n+1}$g(x_{j-1},i,x_j) for all (x_j,i,x_{j-1}) backward arcs and (x_{j-1},i,x_j) forward arcs.

If x_i has label (x^-_{i-1}, i, ε(x_i)), determine

f'(x_i,i,x_{i-1}) = f(x_i,i,x_{i-1}) - {ε(t) $\Pi_{j=i}^{n+1}$g(x_j,i,x_{j-1})}/ $\Pi_{j=i}^{n+1}$g(x_{j-1},i,x_j) for all (x_j,i,x_{j-1}) backward arcs and (x_{j-1},i,x_j) forward arcs.

Step B3: If i>1, go to x_{i-1}, repeat Step B2.

If i=1, repeat step B2 and go to step 2.

Step 2: Erase all labeling. Go to Routine A with flow f'.

Methodology for maximizing flow along a MAN with gains with priority arcs

The labeling algorithm for finding a maximal flow along a MAN with gains with priority arcs, where arc number i denotes the priority number as well, is given below:

Algorithm 2

Step 1: Set i=1.

Start with a feasible flow f≡0 and with v(f)=v(0)=0.

Step 2: Using Routines A and B of Algorithm 1, label arcs with i^{th} priority arcs only.

Step 3: If i<n, set i=i+1. Go to step 2.

If i=n, stop.

THE SCHEDULING PROCEDURE

The procedure for planning and scheduling a single-stage, multi-item compatible process consists of the sequential applications of the above algorithms. For the application of the MAN with priority arcs model for scheduling the process, MAN=(s,t,V,A,b) is considered. This network will have a specific structure and this network will be referred to as the PROCESSNET. Here,

b(s,i,Sk)=Dki/gk (rounded to the next higher integer)

b(Sk,-,Er)=Akr

For all other (x,i,y) εN, b(x,i,y)=∞.

The scheduling algorithm is explained below:

Step 1: Consider the PROCESSNET.

Periods for the process commencement is given by To=T(k,i)-tki.

Find { Dki ∀ k,i such that T 0 is the same}

Step 2: Using Algorithm 2, determine the optimal flow.

Illustrative Example

Consider a workshop having two similar lathes E1 and E2 which are being used for manufacturing three types of roller sets S1, S2 and S3. Let the time taken for processing each of the rollers sets S1, S2 and S3 using the lathes E1 and E2 (tki) be 10, 15 and 20 weeks respectively.

Assume that both the lathes are capable of manufacturing any of the products and the capacity of each of the lathes (Akr) is 400 units for each of the product during the time horizon and the yield rates to be 85%.

Consider the order position for the two customers C1 and C2 (in the same order of customer priorities) for the three roller sets S1, S2 and S3 to be as shown in Table 1. The delivery time (week number) for the product Sk for customer Ci, that is T(k,i), is given in the brackets. Note that changeover for handling one type of roller to the other type incurs certain time and associated costs. Since the changeover costs increase with the number of set ups, to minimize the costs associated with the set ups, similar types of products should be processed successively so that the total number of changeovers are minimized.

This problem is equivalent to the problem of maximizing the total capacity utilization.

Let us consider the problem of scheduling the workshop during this week (To), say, week number 10. This example illustrates the steps in scheduling for the current period.

Table1. *Order position and delivery schedules for the two customers C1 and C2*

Customer/Product	S1	S2	S3
C1	600	500	300
	(20)	(25)	(30)
C2	400	300	700
	(20)	(25)	(30)

The PROCESSNET for the workshop is as shown in Figure 2.

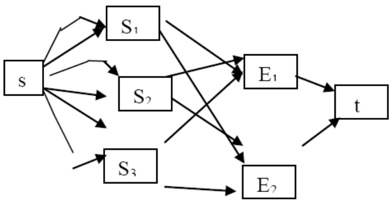

Fig2. *PROCESSNET for the Workshop.*

Applying Step 1 by considering the processing times tkr along the arcs (Sk,Er) and applying the methodology for determining the periods for process commencement as in 4.1, the demand for the product Sk by customer Ci, viz., Dki for the week number 10 will be follows:

D11=600, D12=400, D21=500, D22=300, D31=300 and D32=700.

The yield adjusted order quantity Dki/g_k (rounded to the next higher integer) for the product Sk by the customer Ci, with priority I is given in Table 2.

Table2. *Yield adjusted order position and delivery schedules for the two customers C1 and C2*

Customer/Product	S1	S2	S3
C1	706	589	353
	(20)	(25)	(30)
C2	471	353	824
	(20)	(25)	(30)

As Step 2, applying the Algorithm 2 of maximizing flow for the same multiple arc PROCESSNET the optimal schedule is arrived at. The capacity along the ith arc from s to Sk is the yield adjusted order quantity Dki/g_k (rounded to the next higher integer) for the product Sk by the customer Ci, with priority i. The capacity of the arc (Sk, Er) is considered as Akr which is the capacity of processing product Sk by equipment Er. The capacities of the arcs (Er,t) are considered as infinity. Applying Algorithm 2 of maximizing flow along the MAN with priority arcs, the following maximal flow is obtained:

f* = (706, 94, 589, 211, 353, 447, 400, 400, 400, 400, 400, 400, 1200, 1200).

Thus, the optimal quantities to be supplied to the customers and the corresponding unfulfilled demands (as shown in the brackets) are shown in Table 3. Observe that there are 289 units, 51 units and 174 units of unfulfilled quantities for customer C2 for the products S1, S2 and S3 respectively. These quantities could be rescheduled with customer's concurrence.

Table3. *Optimal supply quantities and the unfulfilled demands for the two customers C1 and C2*

Customer/Product	S1	S2	S3
C1	600	500	300
	(0)	(0)	(0)
C2	111	249	526
	(289)	(51)	(174)

Further Consideration

In case the order books are not full for some of the products in a sequence, we can introduce a dummy customer with least priority having infinite orders, for each product. The scheduling of the orders for the dummy customer brings out the actual product-wise spare capacity. This serves as a guideline to the sales department in procuring further orders for particular products, so that the sequence and in turn the system capacity utilization is maximized.

CONCLUSION

The methodology of the system described above is in general applicable to all single-stage, multi-item compatible production processes where the process can handle different types of products involving changeover costs. This methodology takes into account the yield for the product as well, which reflects the actual quantity of material delivered to the customer and can be compared with the order quantity. Depending on the layout and process flow of the plant, a tailor-made production planning and scheduling system can be formulated using MAN with gains techniques.

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