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Using Other Multi-Attribute Decision Making Techniques to Measure Efficiency When Data Envelopment Analysis Fails

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ABSTRACT

In this article, we show an example of data envelopment analysis that does not provide us with efficiencies based on the same selection of inputs and outputs that were provided by the business management decision maker. Therefore, we select another multi-attribute decision making tool such as TOPSIS, simply modify it to calculate "efficiency", and use entropy weighting method to be able to use all the inputs and outputs in our efficiency decision process.

INTRODUCTION

Here at William and Mary, we teach an Operations Research decision theory class. Topics include analysis of decision under certainty, uncertainty, risk, conflict, and multi-criteria. This article developed out of a class project to illustrate the use of data envelopment analysis to rank order the efficiency of five DMUs with 2 inputs and 3 outputs provided.

DATA ENVELOPMENT ANALYSIS

We start by describing Data Envelopment Analysis (DEA).

Description and Uses

Data envelopment analysis (DEA) is adata input-output driven approach for evaluating the performance of entities called decision makig units (DMUs) that convert multiple inputs into multiple outputs (Cooper, 2000). The definition of a DMU is generic and very flexible so that any entity to be ranked might be a DMU. DEA has been used to evaluate the *performance* or *efficiencies* of hospitals, schools, departments, US Air Force wings, US armed forces recruiting agencies, universities, cities, courts, businesses, banking facilities, countries, regions, SOF air bases, keynodes in networks, and the list goes on. According to Cooper (2000), DEA has been used to gain insights into activities that were not obtained by other quantitative or qualitative methods.

Charnes, Cooper, and Rhodes (1978) described DEA as a mathematical programming model applied to observational data. It provides a new way of obtaining empirical estimates of relationship among the DMUs. It has been formally defined as a methodology directed to frontiers rather than central tendencies.

Methodology

The model, in simplest terms, may be formulated and solved as a linear programming problem (Winston, 1995). Although several formulations for DEA exist, we seek the most straight forward formulation in order to maximize an efficiency of a DMU as constrained by inputs and outputs as shown in equation 1. As an option, we might normalize the metric inputs and outputs for the alternatives if the values are poorly scaled within the data. We will call this data matrix, X, with entries x_{ij}. We define an efficiency unit as $E_i for i=1,2,\ldots,nodes$. We let w_i be the weights or coefficients for the linear combinations. Further, we restrict any efficiency from being larger than one. Thus, the largest efficient DMU will be 1. This gives the following linear programming formulation for single outputs but multiple inputs:

$$Max\ E_i$$

$$subject\ to$$

$$\sum_{i=1}^{n} w_i x_{ij} - E_i = 0, j = 1,2,\ldots \quad (1)$$

$$E_i \leq 1\ for\ all\ i$$

For multiple inputs and outputs, we recommend the formulations provided by Winston (1995) and Trick (2014) using equation (2).

For any DMU_0, let X_i be the inputs and Y_i be the outputs. Let X_0 and Y_0 be the DMU being modeled.

$$\begin{aligned} &Min\ \theta \\ &Subject\ to \\ &\Sigma\lambda_i X_i \leq \theta X_0 \qquad (2) \\ &\Sigma\lambda_i Y_i \leq Y_0 \\ &\lambda_\iota \geq 0 \\ &Non\text{-}negativity \end{aligned}$$

Strengths and Limitations to DEA

DEA can be a very useful tool when used wisely according to Trick (2014). A few of the strengths that make DEA extremely useful are (Trick, 2014): *(1) DEA can handle multiple input and multiple output models; (2) DEA doesn't require an assumption of a functional form relating inputs to outputs; (3) DMUs are directly compared against a peer or combination of peers; and (4) Inputs and outputs can have very different units. For example, X_1 could be in units of lives saved and X_2 could be in units of dollars without requiring any a priori tradeoff between the two.*

The same characteristics that make DEA a powerful tool can also create limitations to the process and analysis. An analyst should keep these limitations in mind when choosing whether or not to use DEA. A few additional limitation include:

- Since DEA is an extreme point technique, noise in the data such as measurement error can cause significant problems.
- DEA is good at estimating "relative" efficiency of a DMU but it converges very slowly to "absolute" efficiency. In other words, it can tell you how well you are doing compared to your peers but not compared to a "theoretical maximum."
- Since DEA is a nonparametric technique, statistical hypothesis tests are difficult and are the focus of ongoing research.
- Since a standard formulation of DEA with multiple inputs and outputs creates a separate linear program for each DMU, large problems can be computationally intensive.
- Linear programming does not ensure all weights are considered. We find that the value for weights are only for those that optimally determine an efficiency rating. If having all criteria weighted (inputs, outputs) is essential to the decision maker then do not use DEA.

Illustrative Example

Consider the following manufacturing process, (modified from Winston, 1995) where we have three DMUs each of which has 2 inputs and 3 outputs as shown in the data table.

Example1. *Manufacturing with DEA that works well*

DMU	Input #1	Input #2	Output #1	Output #2	Output #3
1	*5*	*14*	*9*	*4*	*16*
2	*8*	*15*	*5*	*7*	*10*
3	*7*	*12*	*4*	*9*	*13*

Since no units are given and the scales are similar so we decide not to normalize the data. We define the following decision variables:

t_i = value of a single unit of output of $DMU\ i$, for$i=1,2,3$

w_i= cost orweights for one unit of inputs of $DMU\ i$, for$i=1,2$

$efficiency_i$ = DMU_i=(total value of i's outputs)/(total cost of i's inputs), for $i=1,2,3$

The following modeling assumptions are made:

1. No DMU will have an efficiency of more than 100%.
2. If any efficiency is less than 1, then it is inefficient.
3. We will scale the costs so that the costs of the inputs equals 1 for each linear program. For example, we will use $5w_1+14w_2=1$ in our program for DMU_1.
4. All values and weights must be strictly positive, so we use a constant such as 0.0001 in lieu of 0. This idea from Trick (2014).

To calculate the efficiency of DMU_1, we define the linear program using equation (2) as

Maximize $DMU_{1=}9t_1+4t_2+16t_3$

Subject to

$-9t_1-4t_2-16t_3+5w_1+14w_2\geq 0$

$-5t_1-7t_2-10t_3+8w_1+15w_2\geq 0$

$-4t_1-9t_2-13t_3+7w_1+12w_2\geq 0$

$5w_1+14w_2=1$

$t_i\geq 0.0001,\ i=1,2,3$

$w_i \geq 0.0001,\ i=1,2$

Non-negativity

To calculate the efficiency of DMU_2, we define the linear program using equation (2) as

Maximize $DMU_2 = 5t_1+7t_2+10t_3$

Subject to

$-9t_1-4t_2-16t_3+5w_1+14w_2 \geq 0$

$-5t_1-7t_2-10t_3+8w_1+15w_2 \geq 0$

$-4t_1-9t_2-13t_3+7w_1+12w_2 \geq 0$

$8w_1+15w_2=1$

$t_i \geq 0.0001,\ i=1,2,3$

$w_i \geq 0.0001,\ i=1,2$

Non-negativity

To calculate the efficiency of DMU_3, we define the linear program using equation (2) as

Maximize $DMU_3=4t_1+9t_2+13t_3$

Subject to

$-9t_1-4t_2-16t_3+5w_1+14w_2 \geq 0$

$-5t_1-7t_2-10t_3+8w_1+15w_2 \geq 0$

$-4t_1-9t_2-13t_3+7w_1+12w_2 \geq 0$

$7w_1+12w_2=1$

$t_i \geq 0.0001,\ i=1,2,3$

$w_i \geq 0.0001,\ i=1,2$

Non-negativity

The linear programming solutions show the efficiencies as $DMU_1=DMU_3=1,\ DMU_2= 0.773\ 03.$

Interpretation: DMU_2 is operating at 77.303% of the efficiency of DMU_1 and DMU_3. Management could concentrate some improvements or best practices from DMU_1 or DMU_3 for DMU_2. An examination of the dual prices for the linear program of DMU_2 yields λ_1=0.261538, λ_2=0, and λ_3=0.661538. The average output vector for DMU_2 can be written as:

$$0.261538\begin{bmatrix}9\\4\\16\end{bmatrix}+0.661538\begin{bmatrix}4\\9\\13\end{bmatrix}=\begin{bmatrix}5\\7\\12.785\end{bmatrix}$$

and the average input vector can be written as

$$0.261538\begin{bmatrix}5\\14\end{bmatrix}+0.661538\begin{bmatrix}7\\12\end{bmatrix}=\begin{bmatrix}5.938\\11.6\end{bmatrix}$$

In our data, output #3 is 10 units. Thus, we may clearly see the inefficiency is in output #3 where 12.785 units are required. We find that they are short 2.785 units (12.785-10=2.785). This helps focus on treating the inefficiency found for output #3.

Sensitivity Analysis: Sensitivity analysis in a linear program is sometimes referred to as "what if" analysis. Let's assume that without management engaging some additional training for DMU_2 that DMU_2 output #3 dips from 10 to 9 units of output while the input 2 hours increases from 15 to 16 hours. We find that these changes in the *technology coefficients* are easily handled in resolving the LPs. Since DMU_2 is affected, we might only modify and solve the LP concerning DMU_2. We find with these changes that *DMU*'s efficiency is now only 74% as effective as DMU_1 and DMU_3.

Thus, we see that data envelopment analysis has the potential to provide excellent insights to managers.

Example2. *When DEA fails to match needs*

DMU Departments	Inputs # Faculty	Input 2 # Majors	Outputs Student credit hours	Outputs Number of students	Outputs Total degrees (MS and PhD)
Unit1	25	51	18,341	9,086	**63**
Unit2	15	18	8,190	4,049	**23**
Unit3	10	23	2,857	1,255	**31**
Unit4	33	32	22,277	6,102	**31**
Unit5	**12**	**18**	**6,830**	**2,910**	**19**

The DEA issues are:

- If you set up the linear programs as suggested with the constants strictly greater than 0, using *wi&ti* ≥ 0.0001, using the Trick method (2014), many of the DMU linear programs are infeasible.
- If you allow the constants to be greater than or equal to zero, then in solving the DMU's linear programs some of the inputs and outputs are excluded by the decision variable's value being zero.

Obviously if the decision maker provides inputs and outputs, we assume that all should be considered and used for decision analysis at non-zero levels.

To attempt a fix to this we decide to use another multi-attribute decision making tool to compute the efficiency value.

MULTI-ATTRIBUTE DECISION MAKING

The Technique of Order Preference by Similarity to Ideal Solution (TOPSIS)

TOPSIS was the result of research and work done by Yoon and Hwang (1980). TOPSIS has been used in a wide spectrum of comparisons of alternatives including: item selection from among alternatives, ranking leaders or entities, remote sensing in regions, data mining, and supply chain operations. TOPSIS is chosen over other methods because it orders the feasible alternatives according to their closeness to an ideal solution (Malczewski, 1996). Why TOPSIS? It's strength over other decision making methods is that with TOPSIS, we can indicate which metric (attributes) should be maximized and which should be minimized. In all other methods, everything is maximized,

Napier (1992) provided some analysis of the use of TOPSIS for the department of defense in industrial base planning and item selection. For years, the military used TOPSIS to rank order the systems' request from all the branches within the service for the annual budget review process as well as being taught again in as part of decision analysis. Current work is being done to show the ability of TOPSIS to rank order nodes of a dark or social network across all the metrics of social network analysis.

In a business setting it has been applied to a large number of application cases in advanced manufacturing processes (Argawal et al, 1991; Parken et al, 1999, Parken et al, 1997), purchasing and outsourcing (Kahramanet. Al. 2009; Shyura et al, 2006), and financial performance measurement (Feng and Wang, 2011).

TOPSIS Methodology and Efficiency Modification

We describe the TOPSIS process is carried out through the following steps.

Step 1

Create an evaluation matrix consisting of *m* alternatives and *n* criteria, with the intersection of each alternative and criteria given as x_{ij}, giving us a matrix $(\boldsymbol{X}_{ij})_{mxn}$.

$$D = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ \cdot \\ \cdot \\ \cdot \\ A_m \end{array} \begin{array}{c} \begin{array}{cccccc} x_1 & x_2 & x_3 & \cdot\ \cdot\ \cdot & x_n \end{array} \\ \begin{bmatrix} x_{11} & x_{12} & x_{13} & \cdot\ \cdot\ \cdot & x_{1n} \\ x_{21} & x_{22} & x_{23} & \cdot\ \cdot\ \cdot & x_{2n} \\ x_{31} & x_{32} & x_{33} & \cdot\ \cdot\ \cdot & x_{3n} \\ \cdot & \cdot & \cdot & & \cdot \\ \cdot & \cdot & \cdot & & \cdot \\ \cdot & \cdot & \cdot & & \cdot \\ x_{m1} & x_{m2} & x_{m3} & \cdot\ \cdot\ \cdot & x_{mn} \end{bmatrix} \end{array}$$

Step 2

The matrix shown as ***D*** above then normalized to form the matrix $\boldsymbol{R} = (\boldsymbol{R}_{ij})_{mxn}$, using the normalization method to obtain the entries,

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum x_{ij}^2}}$$

For $i=1, 2\ldots, m;\ j= 1, 2,\ldots n$

Step 3

Calculate the weighted normalized decision matrix. First we need the weights. Weights can come from either the decision maker or by computation.

Step 3a.

Use either the decision maker's weights for the attributes $x_1, x_2, .. x_n$, pairwise comparisons method, or the entropy weighting scheme, as we use here.

$$\sum_{j=1}^{n} w_j = 1$$

The sum of the weights over all attributes must equal one regardless of the weighting method used.

Step 3b.

Multiply the weights to each of the column entries in the matrix from *Step 2* to obtain the matrix, ***T***.

$$T = (t_{ij})_{m\ x\ n} = (w_j r_{ij})_{m\ x\ n},\ i = 1,2,\ldots,m$$

Step 4

Determine the worst alternative (A_w) and the best alternative (A_b): Examine each attribute's column and select the largest and smallest values appropriately. If the values imply larger is better (profit)

then the best alternatives are the largest values and if the values imply smaller is better (such as cost) then the best alternative is the smallest value.

$$A_w = \{\langle \max(t_{ij} \mid i = 1,2,\ldots,m \mid j \in J_- \rangle, \langle \min(t_{ij} \mid i = 1,2,\ldots,m) \mid j \in J_+ \rangle\} \equiv \{t_{wj} \mid j = 1,2,\ldots,n\},$$

$$A_{wb} = \{\langle \min(t_{ij} \mid i = 1,2,\ldots,m \mid j \in J_- \rangle, \langle \max(t_{ij} \mid i = 1,2,\ldots,m) \mid j \in J_+ \rangle\} \equiv \{t_{bj} \mid j = 1,2,\ldots,n\},$$

where,

$J_+ = \{j = 1,2,\ldots n \mid j)$ associated with the criteria having a positive impact, and

$J_{-} = \{j = 1,2, \dots n | j\}$associated with the criteria having a negative impact.

We suggest that if possible make all entry values in terms of positive impacts.

Step 5

Calculate the L2-distance between the target alternative i and the worst condition A_w

$d_{iw} = \sqrt{\sum_{j=1}^{n}(t_{ij} - t_{wj})^2}$,$i=1,2,\dots m$ and the distance between the alternative i and the best condition A_b

$$d_{ib} = \sqrt{\sum_{j=1}^{n}(t_{ij} - t_{bj})^2}, i=1,2,\dots m$$

Where d_{iw} and d_{ib} are L2-norm distances from the target alternative i to the worst and best conditions, respectively.

Step 6

Calculate the similarity to the worst condition:

$$s_{iw} = \frac{d_{iw}}{(d_{iw} + d_{ib})}, 0 \le s_{iw} \le 1, i = 1,2, \dots, m$$

.$S_{iw}=1$if and only if the alternative solution has the worst condition; and

$S_{iw}=0$ if and only if the alternative solution has the best condition.

Step 7

Rank the alternatives according to their value from S_{iw} $(i=1,2,\dots,m)$.

Step 8 Efficiency = $S_{iw}/(Max\ S_{iw})$, $i=1\dots n$

ENTROPY WEIGHTING SCHEME

Shannon and Weaver (1949) proposed the entropy concept and this concept had been highlighted by Zeleny (1982) for deciding the weights of attributes. Entropy is the measure of uncertainty in the information using probability methods. It indicates that a broad distribution represents more uncertainty than does a sharply peaked distribution. To determine the weights by the entropy method the normalized decision matrix we call R_{ij} is considered. The equation used is

$$e_j = -k \sum_{i=1}^{n} R_{ij} \ln(R_{ij})$$

Where $k = 1/ln(n)$ is a constant that guarantees that $0 \le e_j \le 1$. The value of n refers to the number of alternatives. The degree of divergence (d_j) of the average information contained by each attribute can be calculated as:

$d_j=1-e_j$.

The more divergent the performance rating R_{ij}, *for all i& j*, then the higher the corresponding $d_j$the more important the attribute B_j is considered to be.

The weights, equation 3, are found by the equation,$w_j = \frac{(1-e_j)}{\sum(1-e_j)}$. (3)

Let's assume that the criteria were listed in order of importance by a decision maker. Entropy ignores that fact and uses the actual data to compute the weights. Although homeruns might be the most important criteria to the decision maker it might not be the largest weighted criteria using entropy. Using this method, we must be willing to accept these type results in weights.

ILLUSTRATIVE EXAMPLE

We return to example 2 posed earlier which failed to give adequate results using DEA. We used the data and obtained the entropy weights. We used entropy with our data and found the weights, all non-zero, were:

Input 1	0.15284
Input 2	0.13217
Output 1	0.3095
Output 2	0.26007
Output 3	0.14542

Thus, using entropy none of the decision makers inputs or outputs are eliminated from the decision process to calculate efficiency.

Next, we use TOPSIS with our five DMUs, we found these efficiency results:

	TOPSIS – ENTROPY	< - RANK	Efficiency
DMU1	0.843269228	1	1
DMU2	0.281769134	3	0.334139
DMU3	0.07685004	5	0.091133
DMU4	0.0717316791	2	0.850638
	0.184806072	4	0.219154

CONCLUSION

Clearly, DMU 1 is considered 100%. This is because the largest TOPSIS divided by itself is 1. Therefore, one DMU must be 100% in the process as is the process in DEA. We have provided a methods that utilizes all the manager's inputs and outputs in order to rank order entities.

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